Appendix A: Sample of Instrument to Compare Learning Difficulties with Different Forms of Representation

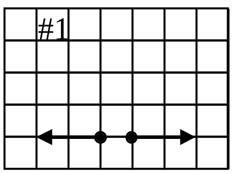
IF YOU WANT A QUESTION GRADED OUT OF THREE POINTS (-1 [MINUS ONE] FOR WRONG ANSWER!!) INSTEAD OF 2.5 POINTS (ZERO FOR WRONG ANSWER) WRITE "3" IN SPACE PROVIDED ON EACH QUESTION.

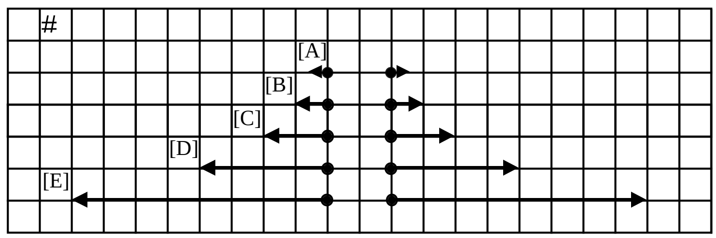
1. When two identical, isolated charges are separated by two centimeters, the magnitude of the force exerted by each charge on the other is eight newtons. If the charges are moved to a separation of eight centimeters, what will be the magnitude of that force now?

   A. one-half of a newton
   B. two newtons
   C. eight newtons
   D. thirty-two newtons
   E. one hundred twenty-eight newtons

   Grade out of three? Write "3" here: ☐

2. Figure #1 shows two identical, isolated charges separated by a certain distance. The arrows indicate the forces exerted by each charge on the other. The same charges are shown in Figure #2. Which diagram in Figure #2 would be correct?





A.
B.
C.
D.
E.

Grade out of three? Write "3" here: ☐

3. Isolated charges q_1 and q_2 are separated by distance r, and each exerts force F on the other. $q_1^{initial} = q_1^{final}$ and $q_2^{initial} = q_2^{final}$; $r^{initial} = 10m$; $r^{final} = 2m$. $F^{initial} = 25N$; $F^{final} = ?$

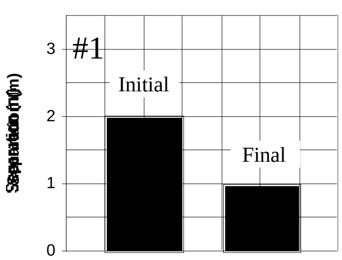
   A. 1 N
   B. 5 N
   C. 25 N
   D. 125 N
   E. 625 N

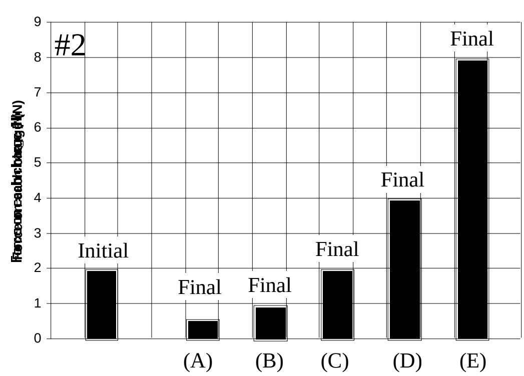
   Grade out of three? Write "3" here: ☐

4. Graph #1 refers to the initial and final separation between two identical, isolated charges. Graph #2 refers to the initial and final forces exerted by each charge on the other. Which bar is correct?

   Grade out of three? Write "3" here: ☐

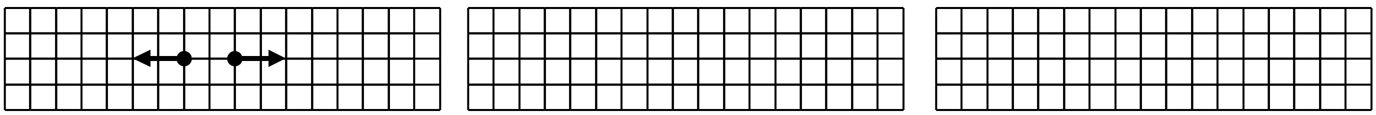
A.
B.
C.
D.
E





Appendix B: Example of Instrument to Diagnose Learning Difficulties with Diagrams and Mathematical Symbols, in the Context of Coulomb's law

1. The diagram on the left shows two isolated particles with equal magnitude charges, along with the electrical forces acting on those particles due to their mutual interactions. The ***same*** charges are to be repositioned in the ***center*** diagram, this time separated from each other by ***half*** the distance that separated them in the diagram on the left. In the diagram on the ***right***, the separation distance is the same as in the center diagram, but the charge on the ***left*** now has ***half*** its original magnitude. Complete the figures in the center and right diagrams to represent the new positions and forces.



Explain for each case:

1) How did you decide where to locate the ***tails*** of the arrows?

2) How did you decide on the ***directions*** of the arrows?

3) How did you decide on the ***lengths*** of the arrows?

2. a) Isolated particles with charges q_1 and q_2 $(q_1 = q_2)$ are separated by distance r, and initially experience mutual interaction forces $\boldsymbol{F_1}^{\boldsymbol{initial}} = 25\text{N}\ \boldsymbol{\hat{i}}$ and $\boldsymbol{F_2}^{\boldsymbol{initial}} = -25\text{N}\ \boldsymbol{\hat{i}}$, where $\boldsymbol{\hat{i}}$ represents a unit vector; $r^{initial} = 10\text{m}$. The particles are repositioned so $r^{final} = 0.2\ r^{initial}$;

$\boldsymbol{F_1}^{\boldsymbol{final}} = ?$

$\boldsymbol{F_2}^{\boldsymbol{final}} = ?$

a) Isolated particles with charges q_1 and q_2 $(q_1^{initial} = q_2^{initial})$ are separated by distance r, and initially experience mutual interaction forces $\boldsymbol{F_1}^{\boldsymbol{initial}} = 25\text{N}\ \boldsymbol{\hat{i}}$ and $\boldsymbol{F_2}^{\boldsymbol{initial}} = -25\text{N}\ \boldsymbol{\hat{i}}$, where $\boldsymbol{\hat{i}}$ represents a unit vector; $r^{initial} = 10\text{m}$. The particles are repositioned so $r^{final} = 0.2\ r^{initial}$, but the magnitude of ***one*** charge is now cut in half: $q_1^{final} = 0.5 q_1^{initial}$; $q_2^{final} = q_2^{initial}$.

$\boldsymbol{F_1}^{\boldsymbol{final}} = ?$

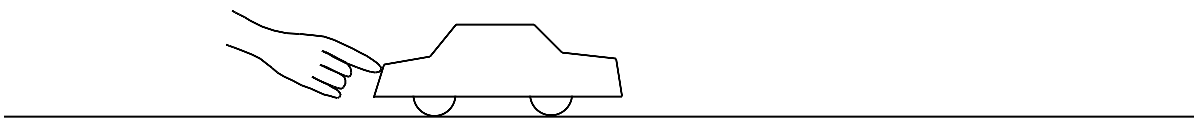
$\boldsymbol{F_2}^{\boldsymbol{final}} = ?$

Explain in detail how you obtained your answers in both cases (a) and (b).

Appendix C: Curricular Materials to Address Learning Difficulties with Diagrammatic Representation of Forces

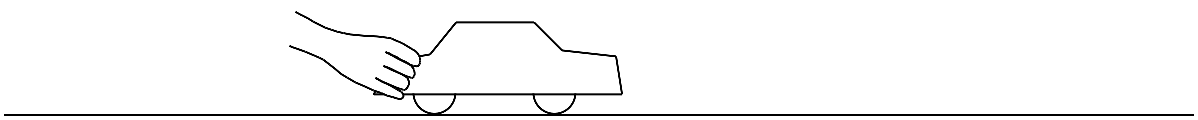
Recall Newton's 1st law of motion: An object at rest tends to remain at rest unless acted on by a **force**. But what exactly is a force? The simplest way to think of force is as a push or pull on an object. If an object is being pulled or pushed, then whatever is doing the pushing or pulling on the object is applying a force to that object.

1)



a) Imagine that you are <u>pushing</u> a toy car as shown above, which causes the car to slowly start moving forward. Draw an arrow pointing in the direction you are pushing; label this arrow "**A**."

b) Suppose you now want to make the toy car speed up ***more quickly*** than in (a). Draw *another* arrow, near the first one, to represent this push. Label it "**B**." Draw the two arrows in (a) and (b) so that the ***longer*** arrow corresponds to the ***harder*** push.



Now imagine that you are gently <u>pulling</u> the car instead, as shown above. Your pull now causes the car to slowly start moving backward.

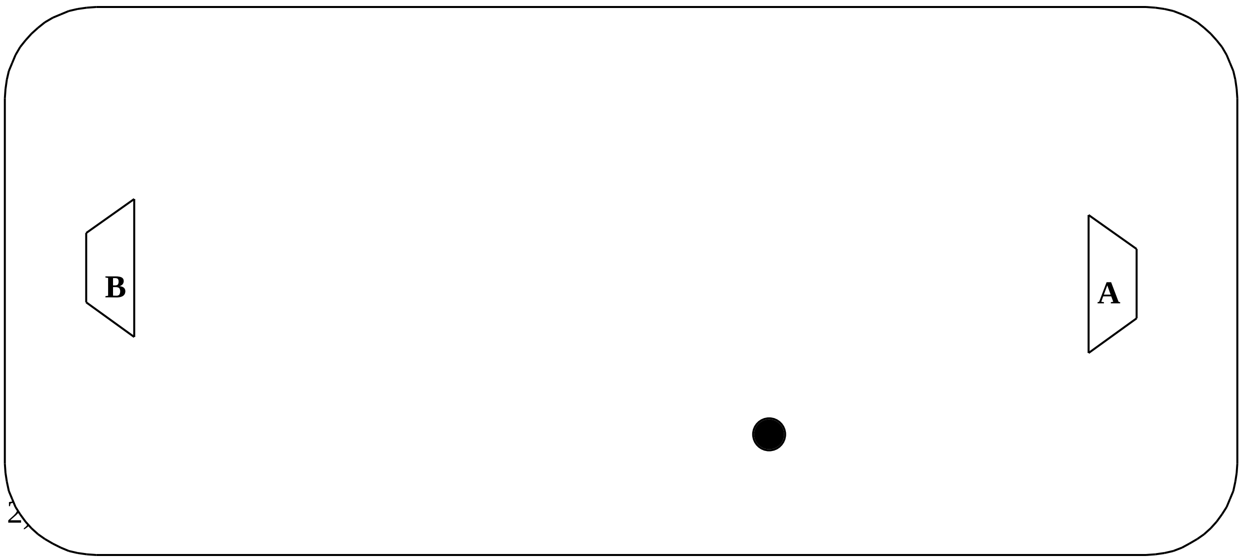
c) Draw an arrow pointing in the direction you are pulling; label this arrow "**C**."

d) Suppose you now want to make the toy car speed up ***more quickly*** than in (c). Draw *another* arrow, near the first one, to represent this pull. Label it "**D**." Again, draw the two arrows in (c) and (d) so that the ***longer*** arrow corresponds to the ***harder*** pull.

You now have a scheme by which you can represent forces (pushes or pulls) in any situation! Forces can be represented by arrows because the arrows themselves represent the two important characteristics of a force:

i) First, the length of the arrow represents the strength or ***magnitude*** of the force. Longer arrows represent stronger forces.

ii) Second, the arrow must point in some ***direction***. The direction in which an arrow points indicates the direction in which the force is acting.



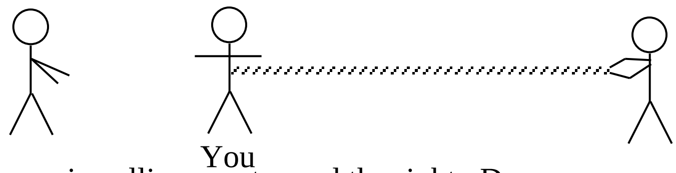
a) Draw an arrow on the puck indicating a force which will send it directly into net A (above right). Label this arrow "**A**."

b) Draw a second arrow on the puck indicating a force which will send it into net B (above left) even faster than force "**A**" sent it into net A. (Don't forget to think about the length of the arrow!) Label this new arrow "**B**."

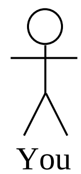
c) Draw an arrow on the puck indicating a force which will cause the puck to miss both nets. Label this arrow "**C**."

As you work through the rest of these worksheets, the phrase "force exerted ***by*** one object ***on*** another object" will appear often. Here's what that means. By "exerts,"it is meant that there is a force by one object on the other object. When we mention the force exerted ***by*** an object, that object is the source of the force which can act on other things. When we mention the force ***on*** another object, this object is the one experiencing the force.

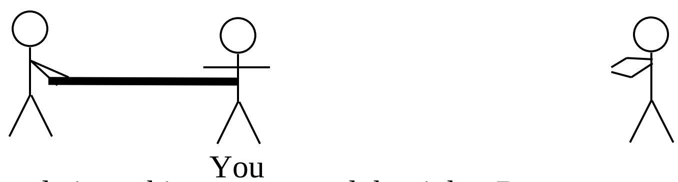
3) Let's make a distinction between a "push" force and a "pull" force.   Imagine that you are tied to one end of a rope, and someone else is pulling on the other end of the rope.



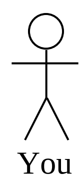
a) The person holding the rope is pulling you toward the right. Draw an arrow that represents the force exerted ***by*** that other person ***on*** you.



On the other hand, imagine that you are being pushed toward the right by a pole held by someone else.



b) The person holding the pole is pushing you toward the right. Draw an arrow that represents the force exerted ***by*** that person ***on*** you.



c) Are the forces exerted ***by*** the people in (a) and (b) ***on*** you in the same direction?

d) Would it be OK to draw the exact same picture for answers (a) and (b)? ***Explain your reasoning.***

Should you draw the same thing for (a) and (b)? YES, that's correct! Since pushes and pulls are both forces, and they both have the same effect, they can be represented by the same arrows. For clarity, you should always place the tail of your arrow on the object which is experiencing such a push or pull.

Appendix D: Curricular Materials to Address Learning Difficulties with Gravitation

Newton's Law of Universal Gravitation is derived from Newton's three laws of motion. It describes the force on one massive object due to the presence of a second massive object. It is written as:

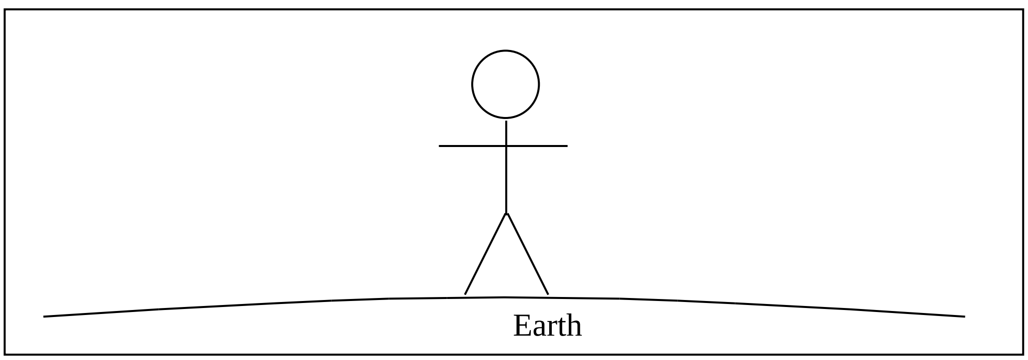
$$\mathbf{F = GM_1M_2/r^2}$$

"M_1" and "M_2" represent the masses of each of the two objects, "r" represents the distance between the two objects, and "F" represents the magnitude (or strength) of the force which one of these massive objects exerts on the other. "G" is a constant (a number which can be looked up if you need a numerical answer) which is always the same, regardless of which two objects are being considered. Note that the force of gravity is a purely attractive force – that means that gravity is always a "pulling" force and never a "pushing" force.

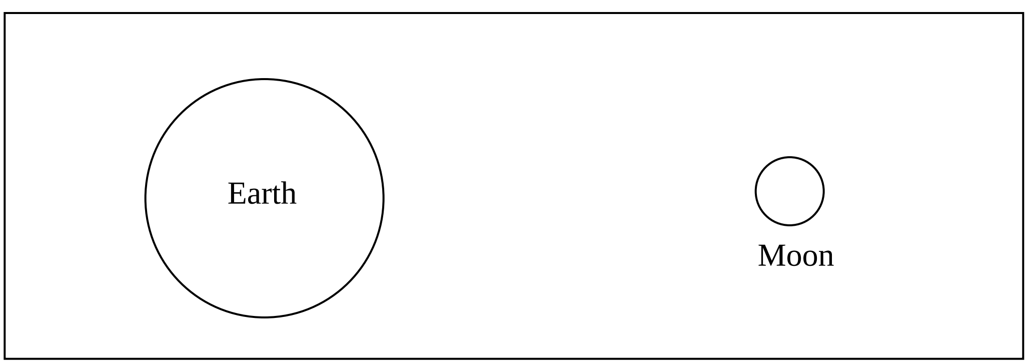
Now let us move into more astronomical settings.

1.

a) In the picture below, a person is standing on the surface of the Earth. Draw an arrow to represent the force exerted ***by*** the Earth ***on*** the person.



b) In the picture below, both the Earth and the Moon are shown. Draw an arrow to represent the force exerted ***by*** the Earth ***on*** the Moon. Label this arrow **(b)**.



c) Now, in the same picture (above), draw an arrow which represents the force exerted ***by*** the Moon ***on*** the Earth. Label this arrow **(c)**. Remember to draw the arrow with the correct length and direction as compared to the arrow you drew in **(b)**.

d) Are arrows (b) and (c) the same size? Explain why or why not.

e) Consider the magnitude of the gravitational force in (b). Write down an algebraic expression for the strength of the force. (Refer to Newton's Universal Law of Gravitation at the top of the previous page.) Use M_e for the mass of the Earth and M_m for the mass of the Moon.

f) Consider the magnitude of the gravitational force in (c). Write down an algebraic expression for the strength of the force. (Again, refer to Newton's Universal Law of Gravitation at the top of the previous page.) Use M_e for the mass of the Earth and M_m for the mass of the Moon.

g) Look at your answers for (e) and (f). Are they the same?

h) (Circle one) When two objects of unequal mass (like the Earth and the Moon) exert gravitational forces on one another, the magnitude of the force exerted ***by*** the more massive object ***on*** the less massive object is [**SMALLER THAN, THE SAME AS, LARGER THAN**] the magnitude of the force exerted ***by*** the less massive object ***on*** the more massive object.

2) In the following diagrams, draw arrows representing force vectors, such that the length of the arrow is proportional to the magnitude of the force it represents. Use the <u>same scale</u> for all your exercises on this page. For example, if a force in diagram (i) has twice the magnitude of a force in diagram (ii), the arrows representing these forces will also have a length ratio of two to one.

Diagram (i): In this figure, two equal spherical masses (mass = "M") are shown. Draw the vectors representing the gravitational forces the masses exert on each other. Draw your *shortest* vector to have a length equal to *one* of the grid squares.

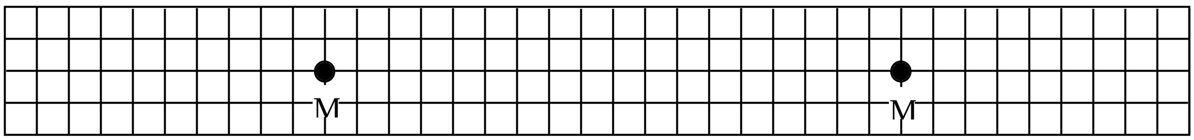


Diagram (ii): Now, one of the spheres is replaced with a sphere of mass 2M. Draw a new set of vectors representing the mutual gravitational forces in this case.

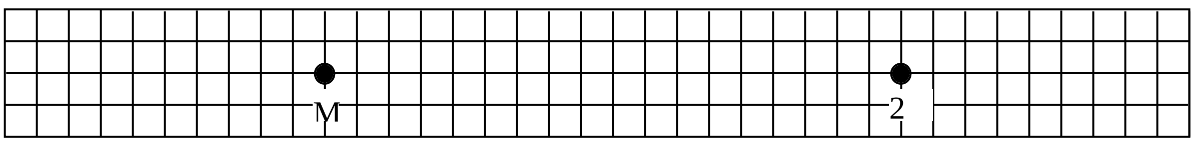
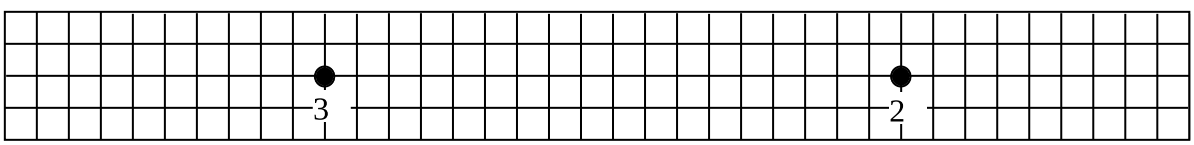


Diagram (iii): In this case, the spheres have masses 2M and 3M. Again, draw the vectors representing the mutual gravitational forces.



a) (Circle one) When the mass of an object is doubled, the magnitude of the gravitational forces exerted ***by*** that mass ***on*** the other mass **[INCREASES, REMAINS THE SAME, DECREASES].**

b) (Circle one) When the mass of an object is doubled, the magnitude of the gravitational forces exerted ***by*** the "unchanged" mass ***on*** the other mass **[INCREASES, REMAINS THE SAME, DECREASES].**

c) (Circle one) When two objects of unequal mass (like the Earth and the Moon) exert gravitational forces on one another, the magnitude of the force exerted ***by*** the more massive object ***on*** the less massive object is [**SMALLER THAN, THE SAME AS, LARGER THAN**] the magnitude of the force exerted ***by*** the less massive object ***on*** the more massive object.

d) Do your answers for (a)-(c) agree with what you drew in Diagrams (i)-(iii)?